

December, 1996

OSLO-TP 20-96

USITP-96-18

hep-th/9612213

Classical Interactions for Tensionless Strings

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ABSTRACT

Using an “action at a distance” formulation we probe the possible classical interactions for tensionless strings, (the $T \rightarrow 0$ limit of the ordinary bosonic string.) We find $G_{\mu\nu}$ and $B_{\mu\nu}$ type interactions but no dilaton interactions.

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1 Introduction

In this brief note we apply the idea of direct string-string interactions to tensionless strings. This approach was used for particle interactions by Feynman and Wheeler [1], and for strings by Kalb and Ramond [2].

The tensionless strings discussed in this note are the $T \rightarrow 0$ limit of ordinary (bosonic) strings, with essentially the same relation to tensile strings as massless particles have to massive ones. Their relation to the tensionless “non critical” $6D$ strings recently discussed in the context of M -theory, [3]-[5], is difficult to judge since the dynamics of the $6D$ -strings is not yet known. The fact that we find possible classical graviton interactions, and that the non-critical strings should not couple to gravitons, speaks against such a relation.

In the search for a relativistic theory of gravitation, it was early realized that gravitation cannot be mediated by a scalar field only. A heuristic way of understanding the problems with such an approach is as follows: The coupling of a scalar (dilaton) field $\phi(X)$ to a massive particle will have the form

$$S_I = m \int d\tau e^\phi \sqrt{-\dot{X}^\mu \dot{X}^\nu \eta_{\mu\nu}}, \quad (1.1)$$

where $\eta_{\mu\nu}$ is the Minkowski metric and dot denotes τ -derivative. This coupling may be interpreted as introducing either an X -dependent mass, or the trace of the metric as an independent degree of freedom. To be able to study massless particles as well, we rewrite (1.1) on a first order form

$$\tilde{S}_I = -\frac{1}{2} \int d\tau \left(g^{-1} e^{2\phi} \dot{X}^2 - g m^2 \right), \quad (1.2)$$

where g is an auxiliary “einbein”. In the limit $m \rightarrow 0$, the field redefinitions

$$\hat{g} \equiv g e^{-2\phi}, \quad \hat{\phi} \equiv \phi, \quad (1.3)$$

remove the coupling between the particle and the dilaton field, thus revealing the problem with interaction of a scalar field with a massless particle. From a different point of view this just reflects the conformal invariance of the action for massless particles. For $m = 0$, we may think of the coupling in (1.2) as resulting from a conformal transformation of the X^μ 's and the redefinition (1.3) as the corresponding conformal transformation of g .

The tensionless string, as discussed in e.g [6]-[9], is the analogue of a massless particle in one dimension higher. In fact, in a particular gauge, a tensionless string just describes a set of massless particles moving subject to a constraint. It is thus not surprising that the abovementioned difficulty in coupling a scalar field extends to the tensionless string too. In what follows we investigate the possible (classical) interactions for tensionless strings using the “action at a distance” formalism, pioneered for particles in [1] and extended to strings in [2]. We find that tensionless strings couple to symmetric and antisymmetric second rank tensor fields, but not to dilatons.

2 Interactions

In this section we want to discuss the possible interactions of tensionless strings via space-time fields. To begin with we give a brief summary of the corresponding treatment of the tensile string, as presented in [2]. The method used is that of the direct interstring action formalism (see e.g.[1] and references therein).

Let X_a^μ denote the coordinates of string number a in the D -dimensional string target space. The corresponding string world sheet coordinates are $\{\xi_a^i\} \equiv \{\tau_a, \sigma_a\}$. We write the total action S for a collection of interacting tensile strings as $S = S_F + S_I$. The free string action S_F is the sum of the free actions for each individual string, $S_F = \sum_a S_{aF}$, where S_{aF} is written in Nambu-Goto form

$$S_{aF} = T \int d^2 \xi_a \sqrt{-\sigma_a^{\mu\nu} \sigma_{a\mu\nu}}, \quad (2.4)$$

where

$$\sigma_a^{\mu\nu} \equiv \epsilon^{ij} \partial_i X_a^\mu \partial_j X_a^\nu = \dot{X}_a^\mu X_a'^\nu - X_a'^\mu \dot{X}_a^\nu, \quad (2.5)$$

dot represents τ_a derivative and prime represents σ_a derivative. (The expression under the square root in (2.4) is indeed minus the determinant of the induced metric.)

The interaction part S_I is given by³ [2]

$$S_I = \sum_{a < b} \int \int d^2 \xi_a d^2 \xi_b R_{ab}(g; X_{a,b}; \partial_i X_{a,b}). \quad (2.6)$$

³We will write all sums over strings explicitly as Σ .

It is assumed that closed as well as open strings contribute to the sums. The functions $R_{ab} = R_{ba}$ depend on the string coordinates and their derivatives, and g is the string coupling constant. We will assume that the dependence on the derivatives is through $\sigma_{\mu\nu}$, in analogy to the particle case. (The tangent to the world line being replaced by the tangent bi-vector to the world sheet.)

The X^μ -equations of motion found by varying S are

$$2TD_a^\nu\left(\frac{\sigma_{a\mu\nu}}{\sqrt{-\sigma_a^2}}\right) = \sum_{b \neq a} \int d^2\xi_b \Delta_{b\mu} R_{ab}, \quad (2.7)$$

where we use

$$\begin{aligned} \sigma_a^2 &\equiv \sigma_a^{\mu\nu} \sigma_{a\mu\nu}, \\ D_a^\mu &\equiv \epsilon^{ij} \partial_j X_a^\mu \partial_i, \\ \Delta_{a\mu} &\equiv \frac{\partial}{\partial X_a^\mu} - \frac{\partial^2}{\partial \xi_a^i \partial (\partial_i X_a^\mu)}. \end{aligned} \quad (2.8)$$

It can be shown from (2.7) that the R_{ab} 's are invariant under separate reparametrizations of the world sheets, [2]. For open strings (2.7) has to be supplemented by boundary conditions at $\sigma_a = 0, \pi$:

$$2T \frac{\sigma_{a\nu}^\mu}{\sqrt{-\sigma_a^2}} \dot{X}_{a\mu}^{o,\pi} = - \sum_{b \neq a} \int d^2\xi_b \frac{\partial R_{ab}}{\partial X_b^\nu}. \quad (2.9)$$

The expressions for R_{ab} representing graviton, anti-symmetric tensor and dilaton exchange between tensile strings were found in [2] to be

$$\begin{aligned} R_{ab}^G &= g^2 \frac{\sigma_a^{\mu\nu} \sigma_{a\nu}^\rho}{\sqrt{-\sigma_a^2}} \frac{\sigma_{b\mu\alpha} \sigma_{b\rho}^\alpha}{\sqrt{-\sigma_b^2}} G, \\ R_{ab}^B &= g^2 \sigma_a^{\mu\nu} \sigma_{b\mu\nu} G, \\ R_{ab}^\phi &= g^2 \sqrt{-\sigma_a^2} \sqrt{-\sigma_b^2} G, \end{aligned} \quad (2.10)$$

where $G \equiv G(s_{ab}^2)$ represents a Green's function of the appropriate kind and $s_{ab}^2 \equiv (X_a - X_b)^2$.

We now want to repeat the analysis for (bosonic) tensionless strings. There is no formulation of these strings corresponding to the Nambu-Goto

action used above, i.e., without an auxiliary field. The closest we can get is to mimic equation (1.2) for the particle, (without the ϕ -field), and write the tensile string action with an auxiliary field [6],

$$\tilde{S}_a = -\frac{1}{2} \int d^2 \xi_a \left(g_a^{-1} \sigma_a^2 - g_a T^2 \right), \quad (2.11)$$

which leads to the $T \rightarrow 0$ action

$$S_{aF}^0 = -\frac{1}{2} \int d^2 \xi_a \left(g_a^{-1} \sigma_a^2 \right). \quad (2.12)$$

Here the auxiliary field g_a is a scalar density to ensure the $2D$ diffeomorphism invariance of the action. We again want to consider a total action of the form $S = S_F + S_I$, with S_F being the sum of free string actions and S_I representing the interactions between them. This combination should represent a limit of the tensionful expression. We therefore assume the same form for the interaction terms, i.e., with R_{ab} being diffeomorphism invariant and constructed from $\sigma_{\mu\nu}$'s. (The R_{ab} 's were already independent of T). We will keep a dependence on a , as yet unknown⁴, tensionless string coupling constant g' . To ensure invariance of R_{ab} we might contemplate a dependence on g_a . The corresponding possibility for the tensile string would be to include a dependence on the auxiliary metric g_{ij} . This possibility is not considered in [2], but, again, there the string action is the Nambu-Goto action which is diffeomorphism invariant by itself. In case we include a dependence on g_a , its field equation is

$$g_a^{-2} \sigma_a^2 + \frac{\partial}{\partial g_a} \sum_{b \neq a} \int d^2 \xi_b R_{ab} = 0. \quad (2.13)$$

From this we see that such a dependence will take us outside the class of tensionless strings, (where $\sigma_a^2 = 0$), in general. For the relation (2.13) to imply $\sigma_a^2 = 0$, the second term must be proportional to the first. But this means that $R_{ab} \propto \sigma_a^2 \sigma_b^2$, which is unacceptable, since then it will vanish on-shell. We will hence assume that R_{ab} is independent of the g_a 's.

⁴The coupling constant for the tensile string has an expression in terms of the dilaton expectation value, and counts the genus of the Riemann surface describing the interaction. No such interpretation of a tensionless string coupling constant exists. We introduce it here by analogy to the tensile case.

Under the above assumptions, the equations of motion that follow from the X^μ -variation of the total action S are

$$D_a^\nu(g_a^{-1}\sigma_{a\mu\nu}) = \sum_{b \neq a} \int d^2\xi_b \Delta_{b\mu} R_{ab}. \quad (2.14)$$

It serves as a gratifying check that the reparametrization invariance of $R_{ab}(g', X, \partial X)$, (no dependence on g_a), follows from (2.14) when $\sigma_a^2 = 0$. In addition to (2.14) we also find the boundary conditions for open tensionless strings

$$g_a^{-1}\sigma_{a\nu}^\mu \dot{X}_{a\mu}^{0,\pi} = \sum_{b \neq a} \int d^2\xi_b \frac{\partial R_{ab}}{\partial X'^\nu_b}. \quad (2.15)$$

Multiplying both sides of eqn. (2.15) by $X'^{0,\pi}_a$ we find a constraint on the R_{ab} 's, namely that

$$X'^{0,\pi}_{a\nu} \sum_{b \neq a} \int d^2\xi_b \frac{\partial R_{ab}}{\partial X'^\nu_b} = 0. \quad (2.16)$$

We deal with (2.15) and (2.16) as follows: In an orthonormal gauge where $X' \dot{X} = 0$, the l.h.s. of (2.15) is proportional to $\dot{X}^2 X'$. We take as a boundary condition on open tensionless strings $X' = 0$ in this gauge. This requires R_{ab} to vanish on the boundaries too, by (2.15). This is satisfied for the R_{ab} 's constructed below. The equations (2.14) and (2.15) determine the motion, and the dynamics of tensionless strings when the interaction terms R_{ab} are specified. We will next turn our attention to the explicit forms of these functions.

The expression for graviton exchange given in (2.10) for the tensile case is of the form $\sim T_a^{\mu\rho} T_{b\mu\rho}$, where $T_a^{\mu\nu}$ is the *space-time* energy-momentum tensor corresponding to S_{aF} (evaluated on the world sheet). In the present case the space-time energy-momentum tensor is

$$\tilde{T}_a^{\mu\nu} \propto g_a^{-1} \sigma^{\mu\gamma} \sigma_\gamma^\nu = g_a^{-1} \epsilon^{ij} \epsilon^{kl} \gamma_{jl} \partial_i X^\mu \partial_k X^\nu, \quad (2.17)$$

(with γ_{ij} the induced metric). Since the R_{ab} 's should be independent of g_a , we try to construct a tensorial R_{ab} from $g_a \tilde{T}_a^{\mu\nu}$. An analogy to the tensile relation (2.10) would be to choose

$$R_{ab}^G = g'^2 \tilde{T}_a^{\mu\nu} \tilde{T}_{b\mu\nu} g_a g_b G \quad (2.18)$$

as a candidate for the gravitational string-string interaction. (Here $G(s_{ab}^2)$ is again an appropriate Greens function.) This choice has the wrong mass dimension⁵ and is not diffeomorphism invariant, however. Instead we construct the following expression

$$R_{ab}^G = g'^2 \sqrt{\sigma_a^{\mu\nu} \sigma_{a\nu}^\rho \sigma_{b\mu\alpha} \sigma_{b\rho}^\alpha} G. \quad (2.19)$$

This expression *is* reparametrization invariant, has the tensor structure appropriate for graviton-like interaction and is of the right dimension.

The expression for exchange in the Kalb-Ramond sector can be copied directly from eq.(2.10) without any changes

$$R_{ab}^B = g'^2 \sigma_a^{\mu\nu} \sigma_{b\mu\nu} G. \quad (2.20)$$

This expression is reparametrization invariant (and non-zero) also in the present case.

Finally we consider possible scalar interactions. The dilaton interaction in (2.10) is proportional to the products of the square roots of the traced energy-momentum tensors of strings a and b . These are the only scalars that we can form from our σ -building blocks of the right dimension. However, in our case these traces are proportional to σ_a^2 and thus vanish, a sign of the space-time conformal invariance of the tensionless strings. We conclude that dilaton interactions are absent.⁶

3 Conclusions

Our results are that tensionless strings may couple to gravitons and antisymmetric tensor fields. This is in good agreement with the possible background geometry terms that we may write down for a tensionless string. Using an equivalent formulation of the tensionless string [7]

$$S^0 = \int d^2\xi V^i V^j \partial X^\mu \partial X^\nu \eta_{\mu\nu}, \quad (3.21)$$

⁵The mass dimension of g' is taken to be 1, as for the tensile string.

⁶We note that a decoupling of the dilaton from supersymmetric non critical strings, under certain circumstances, has been discussed in the context of tensionless strings in M -theory [11].

where the V^i 's are $2D$ vector density fields, we see that the corresponding σ -model action is

$$\tilde{S}^0 = \int d^2\xi \left(V^i V^j \partial X^\mu \partial X^\nu G_{\mu\nu}(X) + \epsilon^{ij} \partial X^\mu \partial X^\nu B_{\mu\nu}(X) \right). \quad (3.22)$$

Here $G_{\mu\nu}$ and $B_{\mu\nu}$ are the background graviton and antisymmetric tensor fields, respectively. Now, the dilaton is related to the trace of $G_{\mu\nu}$ and would couple as $e^\phi \eta_{\mu\nu}$, just as described for the particle in the introduction. Again, as for the massless particle, (3.21) has space time conformal invariance⁷ and such a coupling may be reabsorbed via a conformal transformation of V^i . In the σ -model corresponding to (3.22) for the tensile case, the dilaton enters in two ways: Through a coupling to the $2D$ -curvature scalar R^2 and through an ambiguity in $G_{\mu\nu}$, (string metric vs Einstein metric, e.g.). The first coupling is not available to us since we do not have a $2D$ metric and representing the Euler characteristic in terms of (an integral of) V^i 's seems impossible. In any case it arises as a one-loop effect, and our discussion is purely classical. The second coupling was discussed above.

Couplings in the σ -model action for tensionless strings have previously been discussed in [10], where space-time was restricted to four dimensions. There it was found that (in a special gauge) the $B_{\mu\nu}$ field could be eliminated by a world sheet reparametrization. This result crucially depends on the space time dimension, and cannot be true in general.

4 Acknowledgements

We have benefitted from discussions with M.Roček, P.Saltsidis and B.Sundborg. B.J. thanks ITP, University of Stockholm for hospitality. B.J. also acknowledges partial support from NFR under grant no 110945/432. U.L. acknowledges partial support from NFR under grant no 4038-312 and from NorFA under grant no 96.55.030-O.

⁷This can be extended to invariance of (3.22) under conformal isometries.

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